Chapter 10

TRANSVERSE COUPLED BUNCH INSTABILITIES

10.1 RESISTIVE WALL

If there are M identical equally spaced bunches in the ring, there are $\mu = 0,\ \cdots,\ M{-}1$ transverse coupled modes when the center-of-mass of one bunch leads its predecessor by the betatron phase of $2\pi\mu/M$. The transverse growth rate for the μ-th coupled-bunch mode is exactly the same as the formula in Eq. (9.46) except for the replacement of ω_p by $\omega_q = (qM{+}\mu)\omega_0 + \omega_\beta + m\omega_s$; i.e.,

$$\frac{1}{\tau_{m\mu}} = -\frac{1}{1{+}m}\frac{eMI_bc}{4\pi\nu_\beta E_0}\frac{\sum_q \mathcal{R}e\, Z_1^\perp(\omega_q)h_m(\omega_q{-}\chi/\tau_L)}{B\sum_q h_m(\omega_q{-}\chi/\tau_L)}\ , \tag{10.1}$$

where the bunching factor $B = M\tau_L/T_0$ has been used, $\chi = \omega_\xi\tau_L$ is the chromaticity phase shift across the bunch of full length τ_L and T_0 is the revolution period.

A most serious transverse coupled-bunch instability that occurs in nearly all storage rings is the one driven by the resistive wall [1]. Since $\mathcal{R}e\, Z_1^\perp \propto \omega^{-1/2}$ and is positive (negative) when ω is positive (negative), a small negative frequency betatron line, which acts like a narrow resonance, can cause coupled-bunch instability. Take, for example, the Tevatron in the fixed target mode, where there are $M = 1113$ equally spaced bunches. The betatron tune is $\nu_\beta = 19.6$. The lowest negative betatron frequency line is at $(qM{+}\mu)\omega_0 + \omega_\beta = -0.4\omega_0$, for mode $\mu = 1093$ and $q = -1$. The closest damped

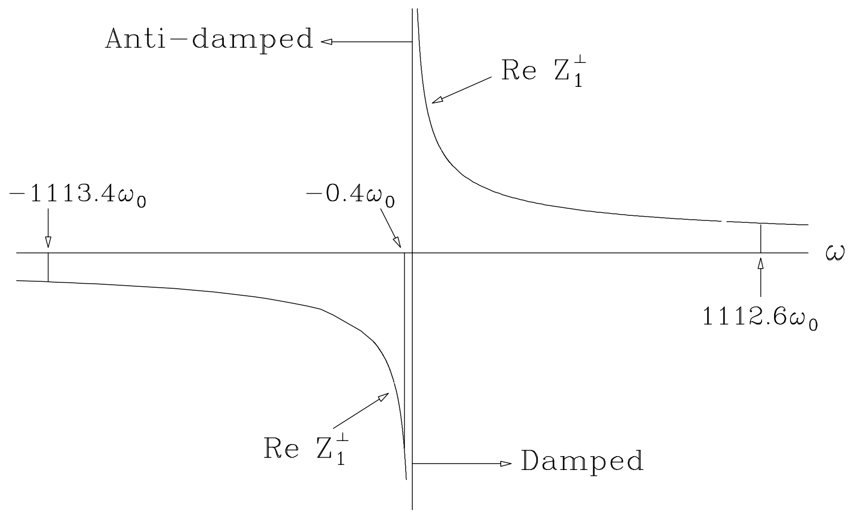

Figure 10.1: The $-0.4\omega_0$ betatron line in the Tevatron dominates over all other betatron lines for $\mu = 1093$ mode coupled-bunch instability driven by the resistive wall impedance.

betatron line ($q = 0$) is at $(1113-0.4)\omega_0$, but $\mathcal{R}e\, Z_1^\perp$ is only $-\sqrt{0.4/1112.6}$ the value at $-0.4\omega_0$. The next anti-damped betatron line ($q = -2$) is at $-1113.4\omega_0$, with $\mathcal{R}e\, Z_1^\perp$ equal to $\sqrt{0.4/1113.4}$ the value at $-0.4\omega_0$. This is illustrated in Fig. 10.1. Thus it is only the $-0.4\omega_0$ betatron line that dominates. From Eq. (10.1), the growth rate for this mode can therefore be simplified to

$$\frac{1}{\tau_{m\mu}} \approx -\frac{1}{1+m}\frac{eMI_bc}{4\pi\nu_\beta E_0}\,\mathcal{R}e\, Z_1^\perp(\omega_q)F'_m(\omega_q\tau_L - \chi)\ , \tag{10.2}$$

where $\chi = \omega_\xi\tau_L$ and the form factor is

$$F'_m(\omega\tau_L) = \frac{2\pi h_m(\omega)}{\tau_L\displaystyle\int_{-\infty}^{\infty} h_m(\omega)d\omega}\ , \tag{10.3}$$

and is plotted in Fig. 10.2. For zero chromaticity, only the $m = 0$ mode can be unstable because the power spectra for all the $m \neq 0$ modes are nearly zero near zero frequency. Since the perturbing betatron line is at extremely low frequency, we can evaluate the form factor at zero frequency. For the sinusoidal modes, we get $F'(0) = 8/\pi^2 = 0.811$. One method to make this mode less unstable or even stable is by introducing positive chromaticity when the machine is above transition. For the Tevatron, $\eta = 0.0028$, total bunch length $\tau_L = 5$ ns, revolution frequency $f_0 = 47.7$ kHz, a chromaticity of $\xi = +10$

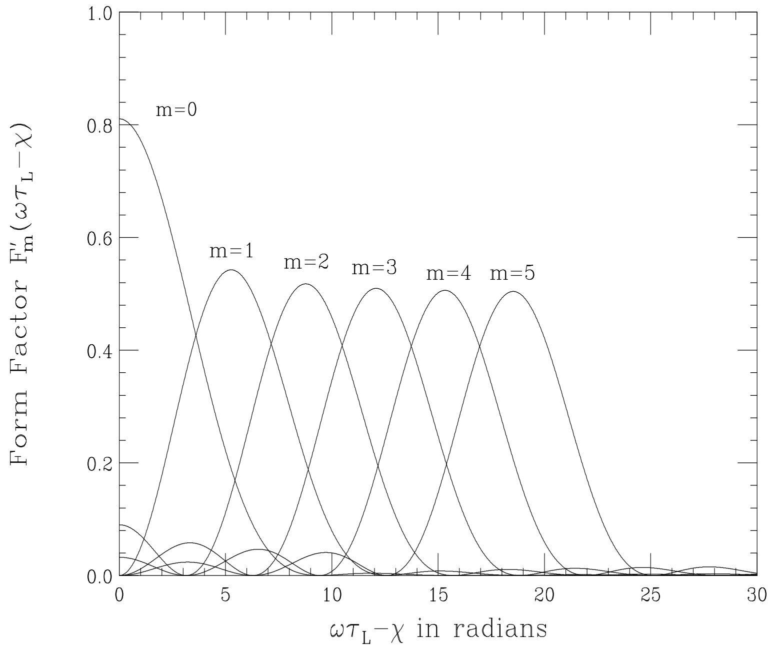

Figure 10.2: Plot of form factor $F'_m(\omega\tau_L-\chi)$ for modes $m = 0$ to 5. With the normalization in Eq. (9.44), these are exactly the power spectra h_m.

will shift the spectra by the amount $\omega_\xi\tau_L = 2\pi f_0\xi\tau_L/\eta = 5.4$. The form factor and thus the growth rate is reduced by more than 4 times. However, from Figs. 6.4 and 9.3, we see that the spectra are shifted by $\omega_\xi\tau_L/\pi = 1.7$ and the $m = 1$ mode becomes unstable. Another method for damping is to introduce a betatron angular frequency spread using octupoles, with the spread larger than the growth rate.

A third method is to employ a damper. Let us derive the displacements of consecutive bunches at a BPM. Suppose the first bunch is at the BPM with betatron phase $\phi_{\beta 0} = 0$; its displacement registered at the BPM is proportional to $\cos\phi_{\beta 0} = 1$. At that moment, the next bunch has phase $2\pi\bar{\mu}/M$ in advance, where $\bar{\mu} = qM + \mu = -20$. When this bunch arrives at the BPM, the time elapsed is T_0/M and the change in betatron phase is $\omega_\beta T_0/M = 2\pi\nu_\beta/M$. The total betatron phase on arrival at the BPM is therefore $\phi_{\beta 1} = 2\pi\bar{\mu}/M + 2\pi\nu_\beta/M = 2\pi(\bar{\mu}-\nu_\beta)/M = (-0.4)2\pi/M$, and the displacement registered is $\cos\phi_{\beta 1}$ When the nth consecutive bunch arrives at the BPM, its phase will be $\phi_{\beta n} = n(-0.4)2\pi/M$. This is illustrated in Fig. 10.3 when the BPM is registering every 20th bunch [2]. What we see at the BPM is a wave of frequency -0.4 harmonic or about 19.1 kHz. Because we know that the bunches follow the pattern of such a slow wave, we

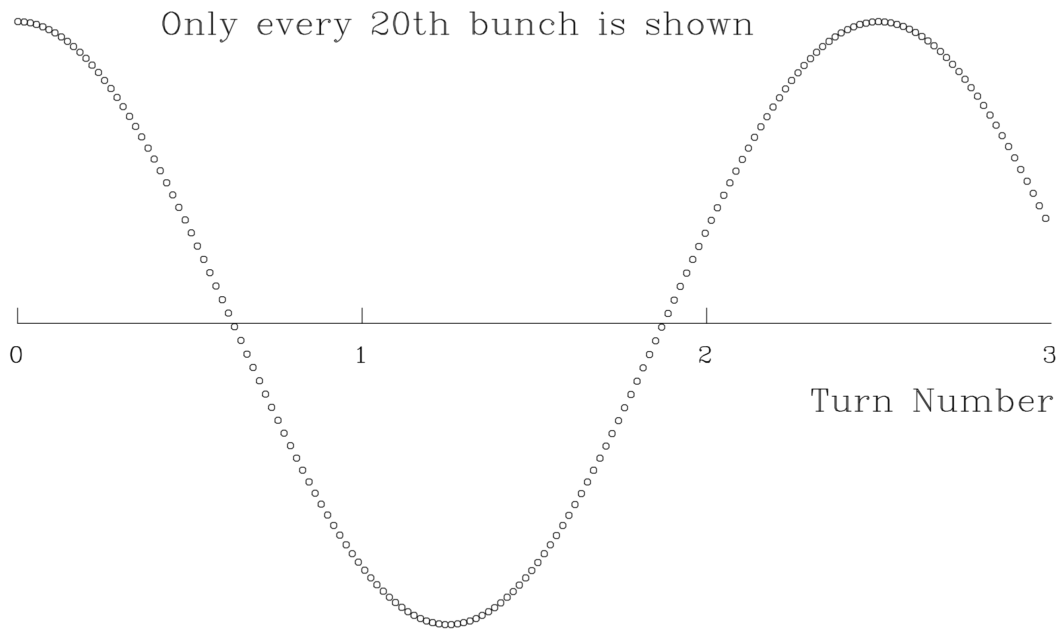

Figure 10.3: Difference signal at a BPM displaying the displacement of every 20th bunch, when the $\mu = 1093$ mode of transverse coupled-bunch is excited by the resistive wall impedance.

only require a very narrow-band feedback system to damp the instability. Usually the adjacent modes $\mu = 1092,\ 1091,\ \cdots$ will also be unstable at the $-1.4\omega_0,\ -2.4\omega_0,\ \cdots$ betatron line; but the growth rates will be smaller.

10.2 NARROW RESONANCES

The narrow higher-order transverse resonant modes of the rf cavities will also drive transverse coupled-bunch instabilities. The growth rates are described by the general growth formula of Eq. (10.1). When the resonance is narrow enough, only the betatron lines closest to the resonant frequency $\omega_r/(2\pi)$ contribute in the summation. The growth rate is therefore given by Eq. (10.2), where two betatron lines are included.

$$\frac{1}{\tau_{m\mu}} \approx -\frac{1}{1+m}\frac{eMI_bc}{4\pi\nu_\beta E_0}[\mathcal{R}e\, Z_1^\perp(\omega_q)F_m'(\omega_q\tau_L-\chi) - \mathcal{R}e\, Z_1^\perp(\omega_{q'})F_m'(\omega_{q'}\tau_L-\chi)]\ , \quad (10.4)$$

where q and q' satisfy

$$\begin{cases} -\omega_r \approx \omega_q = (qM + \mu + \nu_\beta + m\nu_s)\omega_0 \\ \ \ \omega_r \approx \omega_{q'} = (q'M + \mu + \nu_\beta + m\nu_s)\omega_0 \ . \end{cases} \tag{10.5}$$

Similar to the situation of longitudinal coupled-bunch instabilities, mode $\mu = 0$ and mode $\mu = M/2$ if M is even receive contributions from both the positive-frequency side and negative-frequency side. In the language of only positive frequencies, there are the upper and lower betatron side-bands flanking each revolution harmonic line. The lower side-band originates from negative frequency and is therefore anti-damped. For these two modes, both the upper and lower side-bands correspond to the same coupled-bunch mode. If the resonant frequency of the resonance leans more towards the lower sideband, there will be a growth. If the resonant frequency leans more towards the upper side band, there will be damping. This is the Robinson's stability analog in the transverse phase plane. However, sometimes it is not so easy to identify which is the lower sideband and which is the upper sideband. This is because the residual betatron tune $[\nu_\beta]$ or the noninteger part of the betatron tune can assume any value between 0 and 1. If $[\nu_\beta] > 0.5$, the upper betatron sideband of a harmonic will have a higher frequency than the lower betatron sideband of the next harmonic.

There is one important difference between transverse coupled-bunch instabilities driven by the resistive-wall impedance and by the higher-order resonant modes. The former is at very low frequency and therefore the form factor F_1 is close to 1 when the chromaticity is zero. The latter, however, is at the high frequency of the resonances. The form factor usually assumes a much smaller value unless the bunch is very short and we sometimes refer this to "damping" from the spread of the bunch.

This instability can be observed easily in the frequency domain at the lower betatron sidebands flanking the harmonic lines. If a particular lower betatron sideband grows strongly, we subtract the betatron tune ν_β (not $[\nu_\beta]$) to find out which harmonic line it is associated with. Then from Eq. (10.5), we can determine which coupled-bunch mode μ it is. To damp this transverse coupled-bunch instability, one can identify the offending resonant modes in the cavities and damp them passively using an antenna. A tune spread due to the slip factor η or from an octupole can also contribute to the damping. When the above are not efficient enough, a transverse bunch-to-bunch damper will be required. Unlike the situation of the resistive wall, here the damper must be of wide-band.

10.3 EXERCISES

10.1. For the example of resistive-wall driven coupled-bunch instability of the Tevatron at the fixed target mode, try to sum up the contribution for all frequencies for the $\mu = 1093$ mode and compare the result of taking only the lowest frequency line.

10.2. For the same example in Exercise 10.1, compare the growth rates of mode $\mu = 1092,\ 1091,\ \cdots$, with mode 1093. How many modes do we need to include so that the growth rate drops to below 1/4 of that of mode 1093?

10.3. For a narrow resonance that has a total width larger than $2[\nu_\beta]\omega_0$ where $[\nu_\beta]$ is the residual betatron tune and the bunch power spectrum is much wider than the revolution frequency, show that the growth rate is given by

$$\frac{1}{\tau_{m\mu}} \approx \frac{eMI_bc}{4\pi\nu_\beta E_0}\,\frac{h_m(\omega_r-\chi/\tau_L)}{B\sum_{q'} h_m(\omega_{q'}-\chi/\tau_L)}\times$$
$$\times\left\{\mathcal{R}e\, Z_1^\perp[(q_1M-\mu-\nu_\beta)\omega_0-m\omega_s] - \mathcal{R}e\, Z_1^\perp[(q_2M+\mu+\nu_\beta)\omega_0+m\omega_s]\right\}\ , \qquad (10.6)$$

where q_1 and q_2 are some positive integer so that

$$\begin{aligned}(q_1M-\mu-\nu_\beta)\omega_0 &\approx \omega_r\ ,\\ (q_2M+\mu+\nu_\beta)\omega_0 &\approx \omega_r\ .\end{aligned} \qquad (10.7)$$

Such q_1 and q_2 are possible only when $\mu = 0$ or $\mu = M/2$ if M is even. Therefore whether the coupled-bunch mode is stable or unstable depends on whether the resonance is leaning more towards the upper betatron side-band or the lower betatron side-band.

Bibliography

[1] F.J. Sacherer, *Theoretical Aspects of the Behaviour of Beams in Accelerators and Storage Rings*, Proc. First Course of Int. School of Part. Accel., Erice, Nov. 10-22, 1976, p.198.

[2] K.Y. Ng, *Impedances and Collective Instabilities of the Tevatron at Run II*, Fermilab Report TM-2055, 1998.